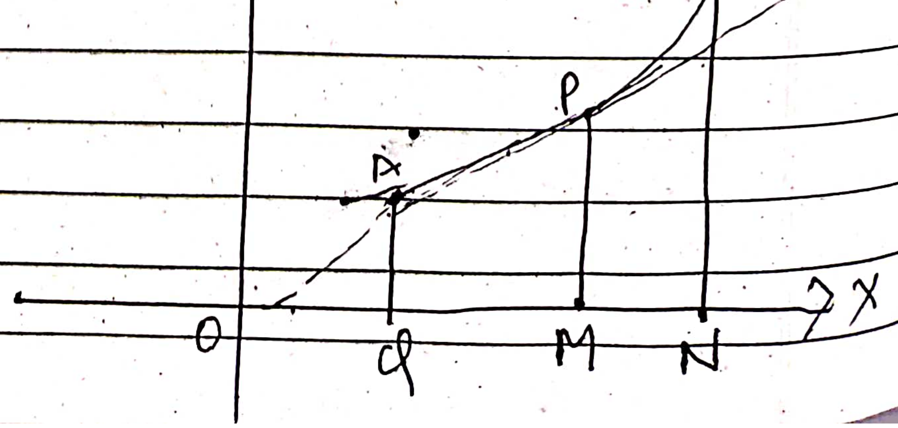
Lengths of Plane Curves

Rectification – The process of determining length of a curve is called rectification.

Length of Plane Curve in case of Rectangular Co-ordinate.

Let P be any point on the curve APB and let (x, y) be its co-ordinates; let a denoted the length of arc AP measured from a fixed point A up to P; then from Differential Calculus we know

$$\frac{ds}{dx} = \sqrt{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}}$$



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Hence $x = \int \sqrt{\left[1 + \left(\frac{dy}{dx}\right)^2\right]} dx$.

where $\frac{dy}{dx}$ is expressed in terms of x

from the equation to the curve.
Thus finding of s is reducible to a question of integration.

If the abscissa of A and B are a and b respectively, then the length s of arc APB is given by

$$s = \int_a^b \sqrt{\left[1 + \left(\frac{dy}{dx}\right)^2\right]} \cdot dx. \quad -(1)$$

If it is convenient to find $\frac{dy}{dx}$ and hence $\frac{dx}{dy}$ in term of y instead of

x from the Equation to the curve we can use formula

$$S = \int_{y_1}^{y_2} \sqrt{\left[1+\left(\frac{dx}{dy}\right)^2\right]}\, dy$$

Where y_1 and y_2 are ordinates of the extreme points on the curve whose length is required.

Again when x and y are given func of single variable parameter φ we have

$$S = \int \left\{\left(\frac{dx}{d\varphi}\right)^2 + \left(\frac{dy}{d\varphi}\right)^2\right\}^{1/2} d\varphi \quad \text{--- } (3)$$